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Key Points:

- Canonical models assume basal heating is a valid approximation in mixed heating modes
- Application of mixed heating changes system energetics and dynamics
- Differing energetics of mixed and basally heated systems may explain the discrepancy between scaling predictions for exoplanets

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The energetics and convective vigor of mixed-mode heating: Velocity scalings and implications for the tectonics of exoplanets

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Abstract The discovery of large terrestrial (~1 Earth mass (M_e) to $< 10\, M_e$) extrasolar planets has prompted a debate as to the likelihood of plate tectonics on these planets. Canonical models assume classic basal heating scaling relationships remain valid for mixed heating systems with an appropriate internal temperature shift. Those scalings predict a rapid increase of convective velocities (V_{rms}) with increasing Rayleigh numbers (*Ra*) and non-dimensional heating rates (*Q*). To test this we conduct a sweep of 3-D numerical parameter space for mixed heating convection in isoviscous spherical shells. Our results show that while V_{rms} increases with increasing thermal *Ra*, it does so at a slower rate than predicted by bottom heated scaling relationships. Further, the V_{rms} decreases asymptotically with increasing *Q*. These results show that independent of specific rheologic assumptions (e.g., viscosity formulations, water effects, and lithosphere yielding), the differing energetics of mixed and basally heated systems can explain the discrepancy between different modeling groups. High-temperature, or young, planets with a large contribution from internal heating will operate in different scaling regimes compared to cooler-temperature, or older, planets that may have a larger relative contribution from basal heating. Thus, differences in predictions as to the likelihood of plate tectonics on exoplanets may well result from different models being more appropriate to different times in the thermal evolution of a terrestrial planet (as opposed to different rheologic assumptions as has often been assumed).

1. Introduction

Over the last decade, discovery of a significant number of large terrestrial extrasolar planets has led to speculation regarding the tectonic regimes these worlds may operate within. Specifically, the interest has been in determining the viability of an Earth-like style of plate tectonics as a function of planetary size. Studies to date have led to seemingly contradictory conclusions with some groups predicting that plate tectonics should dominate [e.g., *Valencia et al.,* 2007; *Valencia and O'Connell*, 2009; *van Heck and Tackley*, 2011; *Tackley et al.,* 2013] and other groups arguing that stagnant-lids will be prevalent [e.g., *O'Neill and Lenardic*, 2007; *Stein et al.,* 2011, 2013].

The connection between mantle convection and planetary tectonics is a complex subject, with many of the studies using differing assumptions and approximations of the underlying rheologies (e.g., different parameterization for lithospheric deformation). It's natural to attempt to reconcile differing model predictions with differing rheologic assumptions. The issue with this approach is that while these processes are clearly important, rheologic properties themselves are not well understood for the Earth, let alone the plethora of recently discovered extrasolar planets. Therefore, any near-term resolution(s) that is(are) formulated around rheologic arguments may depend on poorly known material parameters and, by the definition of rheology, may not be universal (universal in this sense meaning based on conservation laws versus some particular constitutive equation).

There may be a simpler explanation for the discrepancies between various predictions tied to differences in the energetics of dominantly basally, and extrapolations of internally heated systems versus mixed heated systems. A hint that this may be the case comes from the observation that studies predicting the prevalence of plate tectonic regimes [e.g., *Valencia et al.,* 2007; *Valencia and O'Connell*, 2009] assume that the end-member theoretical scalings can be extended to planetary mantles, while those that predict that plate tectonics may be unlikely use the results from numerical convection experiments driven by mixed heating conditions [e.g., *O'Neill and Lenardic*, 2007; *Stein et al.,* 2013]. The intent of this paper is not revisit how parameters may scale with, or be affected by, M_e, but instead to revisit the assumption that classic basal heating scalings can be extended to terrestrial planets with internal heat sources (e.g., radiogenic or tidal). Beyond exoplanetary issues, this question also has broad implications for models of the Earth's thermal history.

2. Classic Scaling Arguments

It is useful to review the classic scaling arguments and the assumptions contained within them when applied to terrestrial planets. Scaling relationships in convective systems are generally expressed in terms of a thermal Rayleigh number (*Ra*):

$$Ra = g\rho\alpha\Delta T d^3/(\kappa\eta) \tag{1}$$

where α is the thermal expansivity, ρ is density, g is gravity, κ is the thermal diffusivity, d is layer (or mantle) depth, and η is the viscosity. ΔT is the reference temperature drop across the system, or the temperature contrast from the base of the convecting layer to the surface (T_s-T_b). Within basally heated systems, classic scaling theory indicates that the V_{rms} (root-mean-square velocity) increases with increasing *Ra*, such that $V_{\mathrm{rms}} \propto Ra^{2/3}$ [e.g., *Schubert et al.,* 2001]. At the simplest level, the V_{rms} is a proxy for the total kinetic energy of the system and provides a measure of the convective vigor.

The V_{rms} is often linked to the shear stress imparted on the lithosphere base by the convecting mantle, given as $\tau = \eta \frac{\partial u}{\partial d}$, where u is the velocity, and a scaling for V_{rms} as a function of *Ra* is used to solve for the velocity. It has been argued that the normal stress (σ) in the lithosphere dominates over basal shear stress τ and that it scales independently of *Ra* [e.g., *Valencia et al.,* 2007; *Valencia and O'Connell*, 2009; *Foley and Bercovici*, 2014]. The argument follows a simple force balance between the shear and normal stresses:

$$\tau L \propto \sigma\delta \tag{2}$$

where L is the convective cell length and δ is the boundary layer thickness. Using V_{rms} and δ consistent with classic basal scaling forms ($\propto Ra^{2/3}$ and $\propto Ra^{-1/3}$, respectively), as well as $\eta \propto Ra^{-1}$ (from inspection of (1)), it is trivial to show that normal stress is independent of *Ra*: $\sigma_{\mathrm{basal}} \propto \frac{Ra^{-1}Ra^{2/3}}{Ra^{-1/3}} \propto C$, where C is a constant value but may scale linearly with M_e [*Valencia and O'Connell*, 2009]. With the added assumption that cell length will always be significantly greater than that of the boundary layer thickness [e.g., *Valencia et al.,* 2007; *Valencia and O'Connell*, 2009; *Foley and Bercovici*, 2014], it follows that σ may dominate in determining whether mantle convection will allow for lithosphere deformation at the level needed to initiate and/or maintain plate tectonics.

A few concepts inherent in the approach above are worth highlighting. The scaling for both shear and normal stress requires a scaling for velocity in terms of a mantle Rayleigh number. The velocity scaling, which was developed for bottom heated systems, is assumed to remain valid for a mixed heating system via a shift in the average internal temperature which accounts for the mantle running hotter due to the decay of radiogenic elements [e.g., *Davies*, 1980; *Schubert et al.,* 2001]. The particular scaling exponent, 1/3 for system heat flux and twice that for velocity, assumes a very high *Ra* such that the upper and lower boundary layers do not interact [*Howard*, 1966]. It additionally assumes that the viscous resistance to convective motions comes from the bulk interior of the convecting system as opposed to the boundary layers themselves. Finally, it is worth noting that for high *Ra* convection in systems were inertial effects are insignificant, which is appropriate for subsolidus convection in the mantles of terrestrial planets, a 1/3 scaling exponent for surface heat flux, and by association a 2/3 exponent for velocity, is an absolute upper bound value [*Chan*, 1971; *Constantin and Doering*, 1999]. Notice that if the scaling exponents fall below those upper bound values, the prediction of normal stress independent of *Ra* breaks down—as is the case if the velocity scaling exponent approaches 1/2 for pure internal heating conditions.

The scaling arguments outlined above depend on the assumption that a convective velocity scaling that is appropriate for a basally heated system can be extended to a mixed system. This is the assumption to be tested in what follows.

3. Mixed-Heating Model Results

To test the scaling relationships between *Ra* and V_{rms} in mixed heating systems, we run suites of numerical experiments using the 3-D community benchmark code CitcomS (version 3.2) [e.g., *Zhong et al.,* 2000, 2008; *Tan et al.,* 2006]. The input parameters *Ra* and non-dimensional internal heating rate (*Q*), defined as:

$$Q = Hd^2/(\kappa\Delta T) \tag{3}$$

where H is the volumetric heating rate, range between 2 orders of magnitude, 1e5–1e7 and 0–200, respectively. Each simulation was run sufficiently long to reach a statistically steady state and then allowed to run

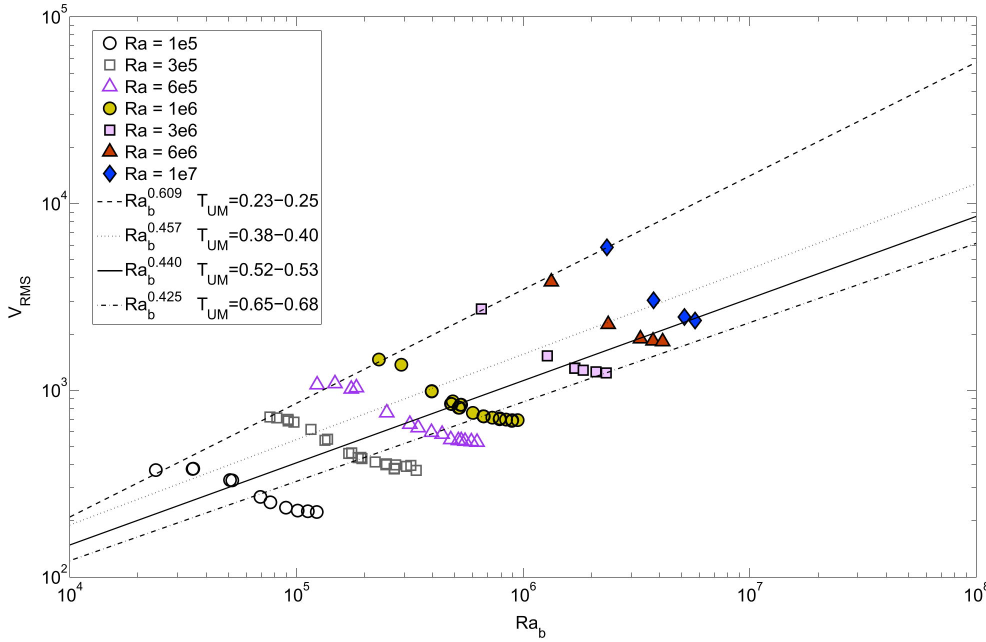

Figure 1. Internal velocities (V_{rms}) versus boundary layer Rayleigh number ($Ra_b = Ra\Delta T_b$), modified after *Weller et al.* [2016]. A range of internal heating rates [0–200] are plotted for each thermal *Ra*. Lines indicate constant Upper Mantle Temperatures (T_{UM}) across variable *Ra* and *Q*. All T_{UM} best fit lines are well described with $R^2 > 0.993$.

between 15% (high *Ra* simulations) and 200% (low *Ra* simulations) longer. Model domains consist of 32, 65, 81, or 128 (low–high *Ra* simulations) grid cell elements in the three primary directions for each of the 12 spherical caps. Boundary conditions are free slip, basal and surface temperatures are fixed ($T_b = 1$; $T_s = 0$), and the core to planetary radius ratio is set at $f = 0.55$. All simulations are of constant viscosity.

Figure 1 illustrates the effects of variable *Q*, through a changing upper boundary layer Rayleigh number Ra_b on the system V_{rms} (modified after *Weller et al.* [2016]). Ra_b is defined as $Ra_b = Ra\Delta T_b$, where ΔT_b is the temperature drop across the upper boundary layer. This accounts for internal heating leading to a larger temperature drop across the upper boundary layer and, hence, a larger negative buoyant potential—this is in keeping with the classic scaling arguments outlined in the introduction which are also formulated in terms of a local boundary layer Rayleigh number [*Howard*, 1966], and it is also in keeping with the manner in which basal heating scalings have been extended to parameterized thermal evolution models for internally heated planets [*Davies*, 1980; *Schubert et al.*, 2001]. Pure basal heating conditions are indicated by a line of constant Upper Mantle Temperature (T_{UM}) of $T_{UM} = 0.23–0.25$. The best fit scaling relationship is indicated by ${Ra_b}^{0.609}$, which is lower than the traditional scaling of exponent of 2/3. However, limiting the parameter range to isolate high *Ra* ($Ra \geq 1e6$) results in fits more closely approaching the 2/3 basal scaling, with a best fit exponent of ${Ra_b}^{0.63}$, indicating that the system is moving toward the theoretical scale at high *Ra* ($Ra \geq e8 - e9$). Below this high Rayleigh number limit, boundary layer interactions lead to a scaling that is different from the classic form [*Moore*, 2008].

While V_{rms} generally increases with increasing system *Ra*, the effects of increasing *Q* are more complex. V_{rms} decreases with increasing *Q* and approaches asymptotes at both high and low levels of internal heating. The introduction of an internal heat source (*Q*) to the system results in a deviation from the basal scaling form. This is shown by best fit lines of constant T_{UM} which reflect scaling exponents that decrease within the parameter space, such that $V_{rms} \propto {Ra_b}^{0.457}–{Ra_b}^{0.425}$ for high levels of internal heating (Figure 1). As noted in *Weller et al.* [2016], simulations compared at equivalent temperatures (which encompass a range of *Q*) are better indicators of system scaling trends than simulations compared at constant input parameters. All T_{UM} best fit lines are well described with $R^2 > 0.993$.

Despite the noted limitations of comparing systems at similar input *Q*, it is still useful to illustrate the effects on V_{rms} of variable *Q* explicitly (Figure 2) given that models which predicted plate tectonics to be less likely for

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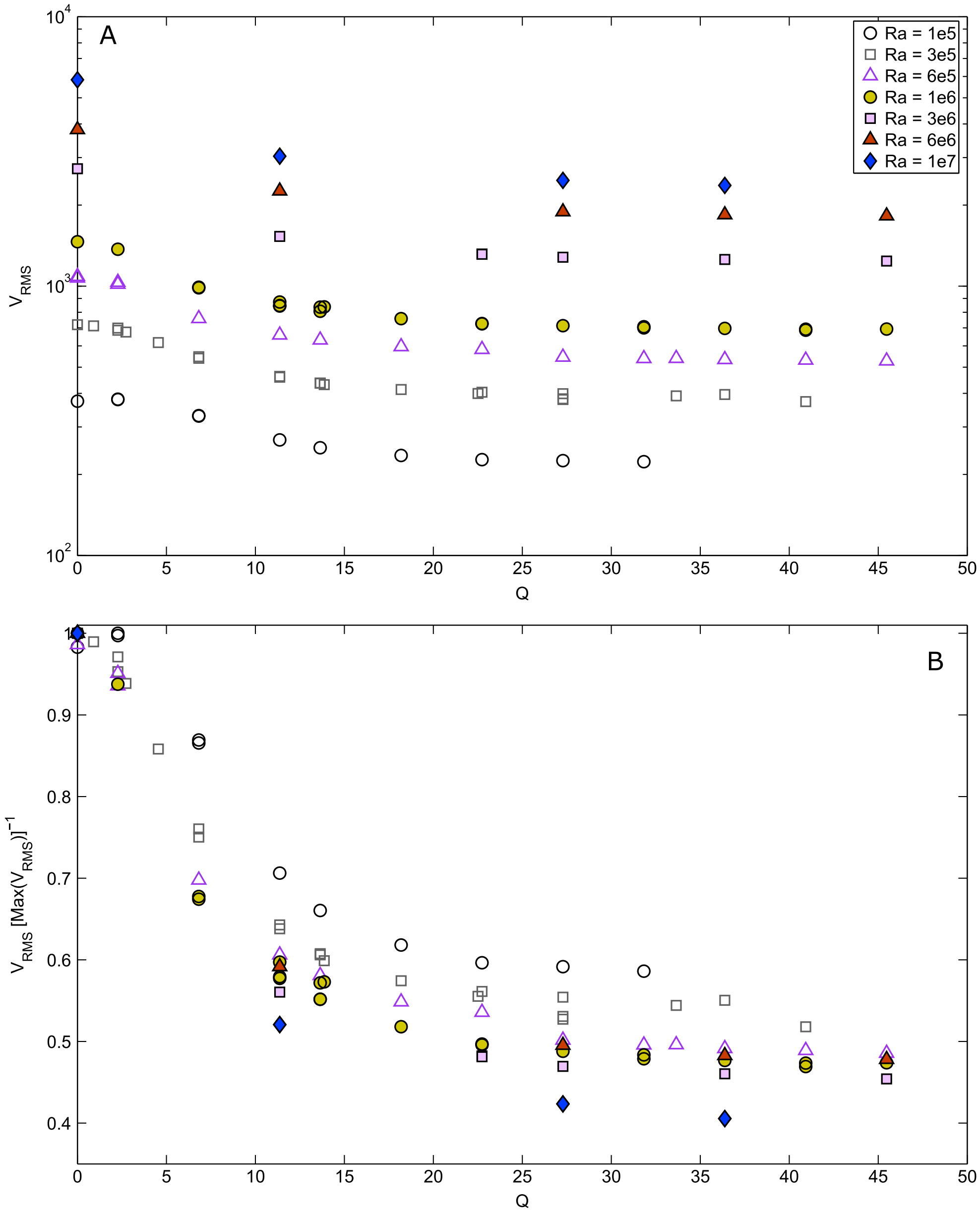

Figure 2. (a) Internal velocities (V_{rms}) versus Internal heating rate (Q); and (b) normalized V_{rms} versus Q. Q values shown are scaled from CitcomS input Q to an effective Q following $Q_{\mathrm{eff}} = \frac{\left(1-f^3\right)}{3}\frac{(1-f)}{f}Q$, where f is the core ratio (following *Weller et al.* [2016]).

massive planets effectively increased internal heating rates as a proxy for planets with greater internal heat sources. The general trend that appears is one of V_{rms} decreasing with Q toward an asymptotic value. Increasing Ra tends to truncate the asymptotic branches where velocity becomes independent of Q (Figure 2a). That is, as Ra increases the regime were V_{rms} decreases with Q extends over a broader Q range. Figure 2b recasts the V_{rms} in a normalized form as a given V_{rms} divided by the maximum V_{rms}, $\frac{V_{\mathrm{rms}}}{\max(V_{\mathrm{rms}})}$, for a fixed Ra. All Ra ranges show similar behaviors as a function of Q, that of decreasing V_{rms}. However, the difference in the maximum to minimum V_{rms} appears to increase as a function of Ra, with $Ra = 1e7$ indicating that $V_{\mathrm{rms}}^{\mathrm{min}} = 0.406\ V_{\mathrm{rms}}^{\mathrm{max}}$, whereas $Ra = 1e5$ indicates that $V_{\mathrm{rms}}^{\mathrm{min}} = 0.586\ V_{\mathrm{rms}}^{\mathrm{max}}$, or $\frac{V_{\mathrm{rms}}^{\mathrm{min}}}{V_{\mathrm{rms}}^{\mathrm{max}}} \propto 1.342\ Ra_b{}^{-0.07}$.

The changing exponent in the V_{rms} scaling (Figure 1) suggests that stress scaling in mixed heating system will-not follow either end-member basal or internal heating trends. In contrast to the classic basal heating scale, mixed heating V_{rms} has a much shallower exponent with Ra_b leading to a normal stress scaling of $\sigma_{\mathrm{mixed}} \propto \frac{Ra_b^{-1} Ra_b^{0.425-0.457}}{Ra_b^{-1/3}} \propto Ra_b^{-0.242} - Ra_b^{-0.201}$ (assuming an end-member basal heating boundary layer scale: $\delta \propto Ra_b^{-1/3}$ which, as previously noted, is an upper limit). If $\delta \propto Ra_b^{-1/4}$, appropriate for the end-member case of pure internal heating, then $\sigma_{\mathrm{internal}} \propto Ra_b^{-0.293} - Ra_b^{-0.325}$. These results suggest that neither basal or internal heating scaling approximations are appropriate to use in mixed heating systems.

4. Discussion and Implication for Planets

Conceptually, reduced velocity, with increasing internal heating (Figure 2), can be explained by the effects of increasing internal temperature on a system with a fixed basal temperature. As Q is increased, the internal temperature increases following a well described scaling form of $Q^{0.75} Ra^{-1}$ [e.g., *Sotin and Labrosse,* 1999; *Moore*, 2008; *Weller et al.,* 2016]. The net effect is to decrease the temperature difference between the interior and base, reducing the basal heat flux. A basal heating Rayleigh number defined for a total system temperature drop is not the appropriate measure for the degree of basal heating the system experiences; the basal heat flux is a model output as opposed to a control parameter. The result is that the total system driving energy does not track internal heating in a one to one manner. As internal heating increases the system can move from a state that is near a basal heating end-member to a state that is near an internally heated end-member. Those end-members do not scale in the same way. One could attempt an empirical regression fit onto models that transition between the two, but the fit would be applied to systems that fundamentally differ in terms of the ratio of driving energy terms (internal heating and heat flowing into the system base).

The above provides an explanation for why mixed heating numerical experiments, with fixed basal temperatures, predict plate tectonics to be less likely for increased levels of internal heating [e.g., *O'Neill and Lenardic*, 2007; *Stein et al.,* 2013], and why results from those experiments are not in line with scaling arguments that assume a single heating mode (e.g., pure basal heating) holds over all ranges of mantle Rayleigh numbers [e.g., *Valencia et al.,* 2007; *Valencia and O'Connell*, 2009; *Foley and Bercovici*, 2014]. With some 20-20 hindsight one can see why in Figure 1 a good regression power law fit could be achieved if experiments with the same internal temperatures were fit, as those cases maintained consistent ratios of internal relative to basal heating (an apples to apples comparison between models that maintain the same heating modes). Even for models with the same heating mode, the scaling of velocity with Rayleigh number is different for pure bottom heating versus mixed heating, which will also feed into different model predictions that stem from scaling arguments based on basal heating versus models that assume a mixed heated system. The latter models lead to the prediction that both shear and normal stresses will decrease with decreasing mantle viscosity and, by association, increasing mantle Rayleigh number. Notice that the explanations above for divergent model predictions are independent of any rheologic assumptions applied to the lithosphere.

There is another factor at work that can cause a mixed heating system to deviate from classic scaling assumptions. As internal heating increases, cell aspect ratios can move toward shorter wavelengths, i.e., smaller L in equation (2) [e.g., *Schubert and Anderson*, 1985]. This can enhance a decrease in convective normal stress with increasing internal heating beyond the decrease that comes from the different scalings for system velocities in mixed versus basally heated systems.

Our findings suggest the potential of plate tectonics on "super-Earths" (or terrestrial planets in general) should be viewed within a temporal and/or internal heat source density framework. In this framework, the predictions developed for exoplanets [*Valencia et al.,* 2007; *Valencia and O'Connell*, 2009], and for general convective systems [e.g., *Foley and Bercovici*, 2014] using basal heating scaling forms (and a formulation for pure internal heating) [*Valencia and O'Connell*, 2009] are not inconsistent with our results, but instead are for special cases of (A) planets that are radiogenically depleted or old enough to have tapped internal heat sources, or sequestered the bulk of them into a nonrecyclable crust while still maintaining sufficient heat flow from the core into the mantle to maintain high values of a bottom heated Rayleigh number; and (B) planets for which the mantle temperature briefly becomes equal to that of the core. Predictions based on mixed heating results are then for systems with high to moderate radiogenics and temperatures. The need to think in a planetary evolution framework suggests that age and compositional constraints (as related to initial heat source density) will be required for probabilistic studies as to the plate tectonic potential of the large set of terrestrial exoplanets that have been, and continue to be, found.

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Erratum

In the originally published version of this article, a typographical error in a reference to a recently published article which contains the data used in this study was discovered. In the Acknowledgments section, Weller et al. [2016] was changed to Weller et al. [2008]. The following have since been corrected and this version may be considered the authoritative version of record.